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BOSTON UNIVERSITY

GRADUATE SCHOOL

Thesis

ALIGNMENT CHARTS AND SOME APPLICATIONS TO PROBLEMS OF PHYSICAL CHEMISTRY

Submitted by

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(A.B., Boston University, 1927)

In partial fulfilment of requirements for

the degree of Master of Arts

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A. Alignment Charts

I. General principles.

Alignment charts are composed of non-uniform scales, sometimes combined with uniform scales, with certain definite geometric relations to each other. If $f(x)$ is a single-valued function, it can be represented on a non-uniform scale. In the case of three variables, three non-uniform scales are so constructed that a transversal or index line cuts them in corresponding values of the variables. The scales are marked with the values of the variables, so that the desired values can be read directly from the chart. These principles have been extensively developed by M. D'Ocagne in his Traité de Nomographie. The types of charts discussed here are used in applications to problems of engineering by Joseph Lipka in his Graphical and Mechanical Computation.

These charts have distinct advantages over contour charts because they are in general more easily constructed and more easily read, interpolation being along a scale instead of between curves. For these reasons they have proved very useful to engineers and to industries where the men who use them have no extensive knowledge of mathematical principles.

The present work discusses charts of various types for three variables, shows how these can be extended to more variables, and how different types can be combined to cover different types of formulae. The applications will be limited to the field of physical chemistry.

II. Oblique and intersecting index lines.

a. Parallel scales.

1. Equations of the form $P(x)+Q(y)=R(z)$

This type also includes equations of the form $P(x)\cdot Q(y)=R(z)$ when we take the logarithms of both sides of the equation, giving $\log P(x)+\log Q(y)=\log R(z)$.

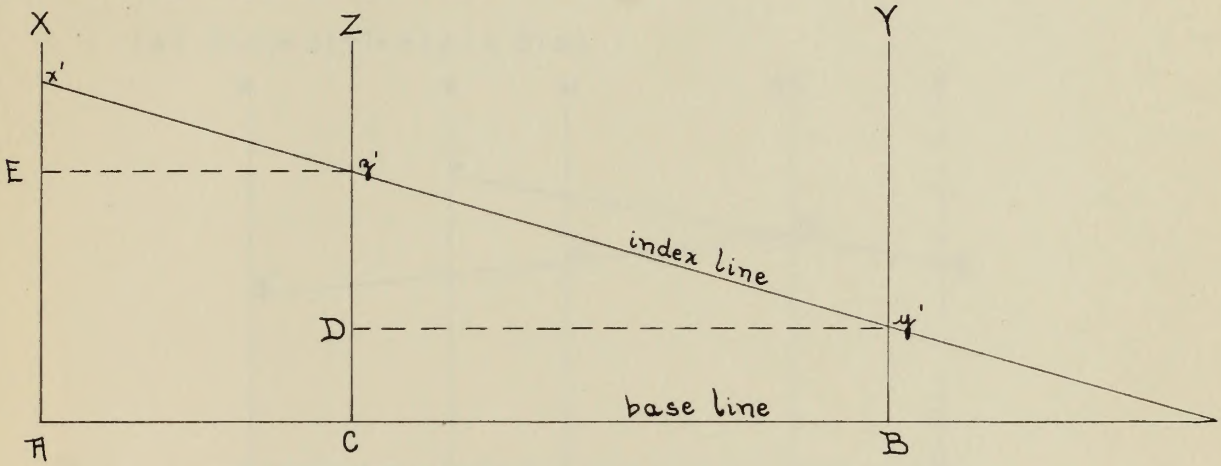

Figure I

Construct three parallel scales on AX, BY, and CZ for $m'P(x)$, $m''Q(y)$, and $mR(z)$ respectively, where m', m'', and m are the moduli of the scales. If $AC:BC=m':m''$, and $m=\frac{m'm''}{m'\ m''}$, then any transversal will cut the scales at corresponding values of the variables.

Proof:

Draw $z'E$ and $y'D$ parallel to the base line.

$$Ex':Dz'=m':m''$$

$$m'P(x')-mR(z'):mR(z')-m''Q(y')=m':m''$$

$$m''m'P(x')-m''mR(z')=m'mR(z')-m'm''Q(y')$$

$$(m'+m'')mR(z')=m''m'P(x')+m'm''Q(y')$$

$$mR(z')=\frac{m'm''}{m'+m''}\left[P(x')+Q(y')\right]$$

Therefore $R(z')=P(x')+Q(y')$, or x', y', and z' are corresponding values of the three variables.

The base line may be drawn at any convenient angle.

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2. Equations of the form $P(x)+Q(y)+R(z) \;......= S(w)$ or $P(x)\cdot Q(y)\cdot R(z)\;......= S(w)$ by taking the logarithms of both members of the equation.

This is merely an extension of the preceding case to. four or more variables.

The case of four variables.

(a) $P(x)+Q(y)+R(z) = S(w)$

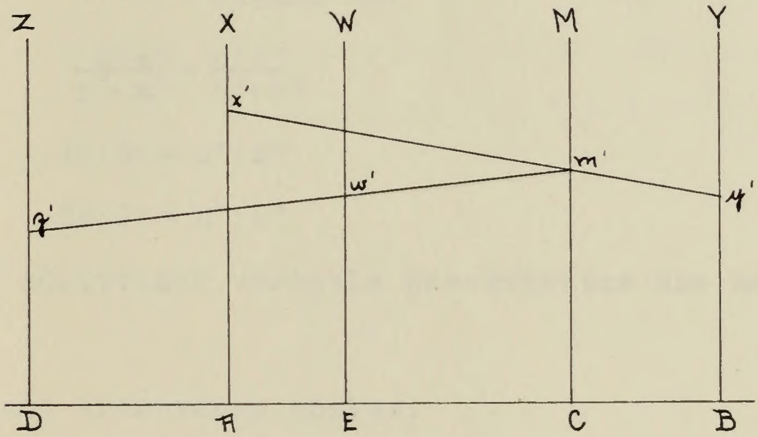


Figure II

Let $P(x)+Q(y) = M$ and construct scales as in the preceding case. Let m', m'', and m be the moduli for $P(x)$, $Q(y)$, and M respectively.

$$\frac{m'm''}{m'+m''} = m \qquad\qquad AC:BC = m':m''$$

$M+R(z) = S(w)$. Let n' and n'' be the moduli for $R(z)$ and $S(w)$.

$$\frac{mn'}{m+n'} = n'' \qquad\qquad CE:DE = m:n'$$

It is evident that index lines through x' and y' and through z' and w' will intersect at m', that is, there is no need of any scale on M. It is also evident that given corresponding values of any three of the variables, the fourth is uniquely determined.

(b) $P(x)+Q(y) = R(z)+S(w)$

Let $P(x)+Q(y) = M$. Then $R(z)+S(w) = M$.

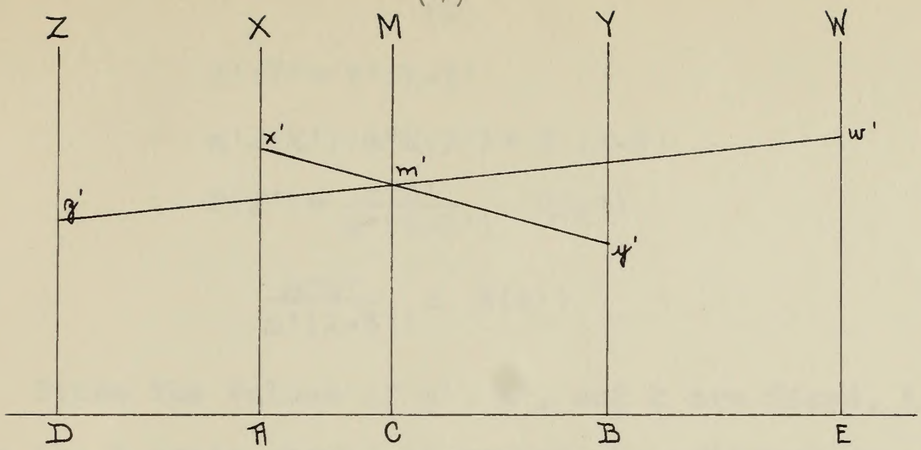


Figure III

$$\frac{m'm''}{m'+m''} = \frac{n'n''}{n'+n''}$$

$$AC:BC = m':m''$$

$$CD:CE = n':n''$$

Each additional variable necessitates one more index line.

b. Parallel and transverse scales.

1. Equations of the form $P(x) = Q(y) \cdot R(z)$, or $P(x) = Q(y)^{R(z)}$ which can be charted as $\log P(x) = R(z) \cdot \log Q(y)$.

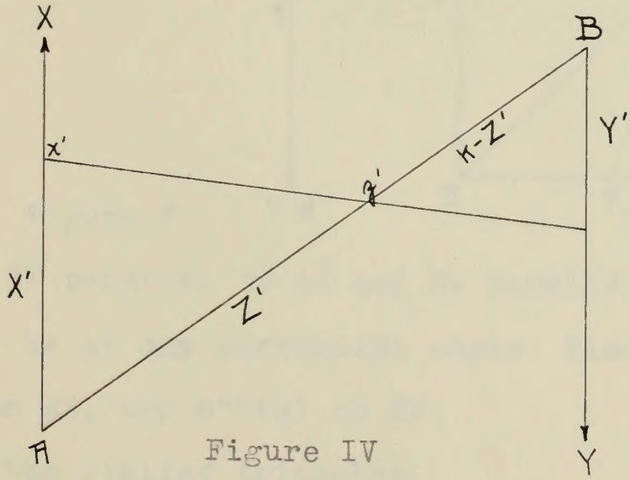


Figure IV

Draw BY parallel to AX and oppositely directed. Place $m'P(x)$ on AX and $m''Q(y)$ on BY. Draw AB and let it equal k.

Let $X' = m'P(x')$

$Y' = m''Q(y')$

$$X':Y' = Z':k-Z'$$

$$m'P(x'):m''Q(y') = Z':k-Z'$$

$$P(x') = \frac{m''Z'}{m'(k-Z')} \cdot Q(y')$$

$$\frac{m''Z'}{m'(k-Z')} = R(z')$$

Since the values of m', m", and k are fixed, this equation shows how the Z scale should be marked. In ordinary practice $R(z)$ will not be complicated enough to make this computation difficult. In any case the simpler of the two functions involved in the product can be taken as $R(z)$.

2. Equations of the form $\frac{P(x)}{Q(y)} = \frac{R(z)}{S(w)}$

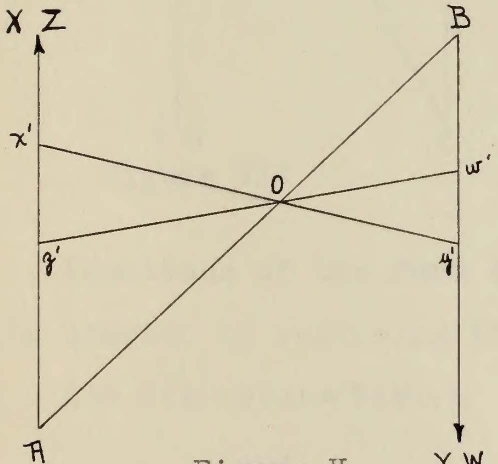


Figure V

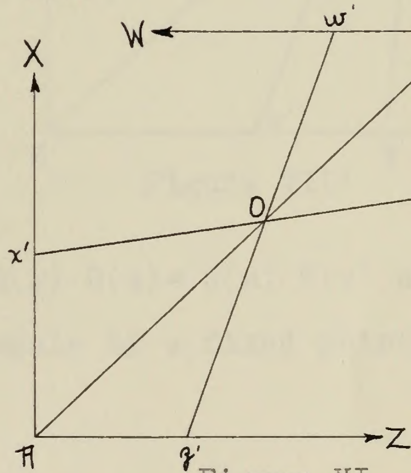


Figure VI

Draw BY parallel to AX and BW parallel to AZ. AX and AZ may coincide or be at any convenient angle. Place $m'P(x)$ on AX, $m''Q(y)$ on BY, $n'R(z)$ on AZ, and $n''S(w)$ on BW.

From the similar triangles,

$$m'P(x'):m''Q(y') = AO:BO$$

$$n'R(z'):n''S(w') = AO:BO$$

$$m'P(x'):m''Q(y') = n'R(z'):n''S(w')$$

If $m':m'' = n':n''$, the scales will be cut in corresponding values of x, y, z, and w.

3. Equations of the form $P(x) \cdot Q(y) \cdot R(z) = S(u) \cdot T(v) \cdot U(w)$

Let $P(x):S(u) = T(v):M$

and

$U(w):Q(y) = R(z):M$

Each form can be charted as in the preceding section, but the m-scale does not need to be graduated.

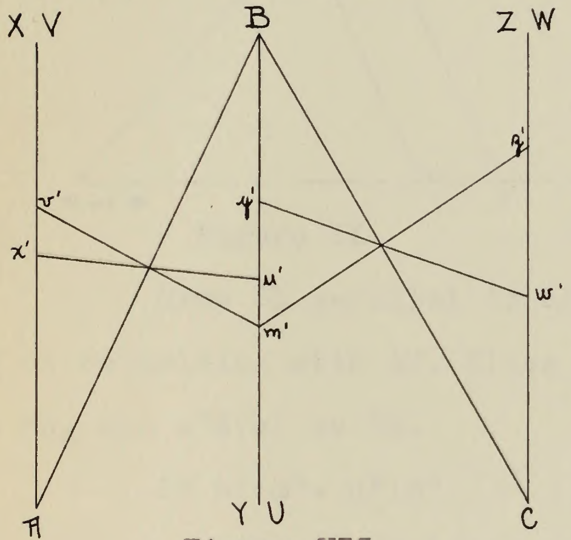


Figure VII

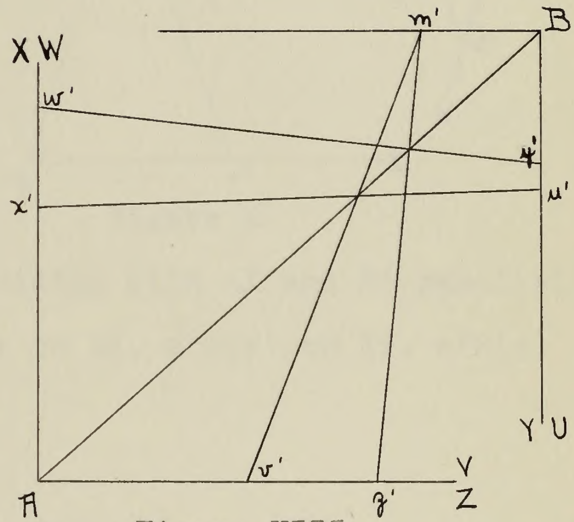


Figure VIII

Equations of the form $P(x) \cdot Q(y) \cdot R(z) = S(u) \cdot T(v)$ can be similarly charted by replacing the w-scale by a fixed point.

Let $P(x):S(u) = T(v):M$

and

$M:R(z) = Q(y):1$

III. Parallel or perpendicular index lines.

1. Equations of the form P(x):Q(y)= R(z):S(w)

(a) Parallel index lines.

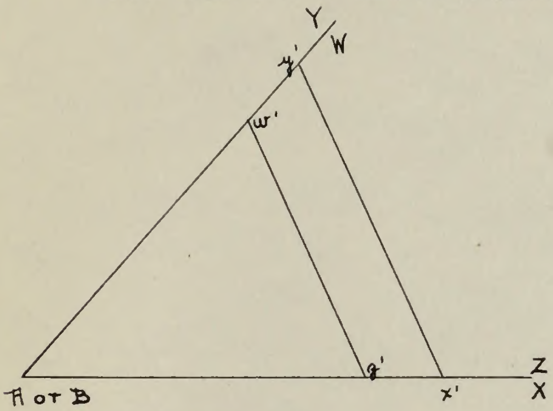


Figure IX

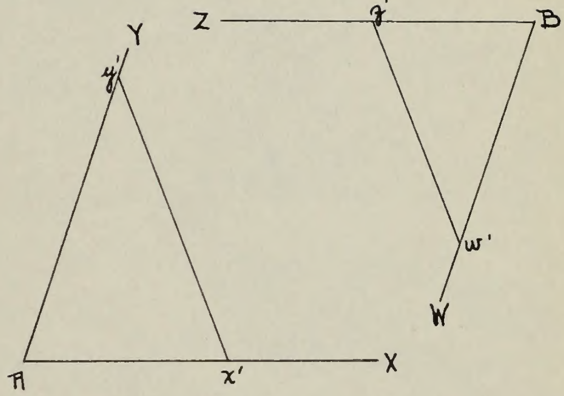


Figure X

Draw BZ parallel to or coinciding with AX and BW parallel to or coinciding with AY. Place m'P(x) on AX, m"Q(y) on AY, n'R(z) on BZ, and n"S(w) on BW.

If m':m"= n':n"

m'P(x'):m"Q(y') = n'R(z'):n"S(w')

This means that the triangles Ax'y' and Bz'w' are similar and therefore the index lines are parallel.

(b) Perpendicular index lines.

If BZ is perpendicular to AX and BW is perpendicular to AY, the triangles Ax'y' and Bz'w' are similar and the index lines are perpendicular.

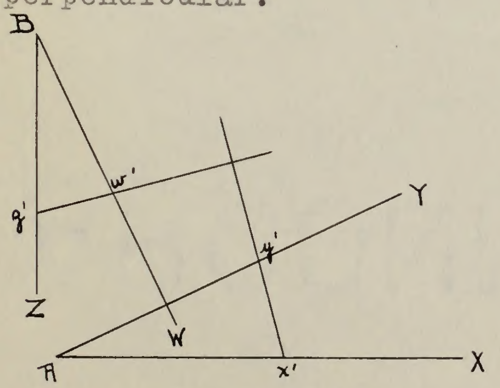


Figure XI

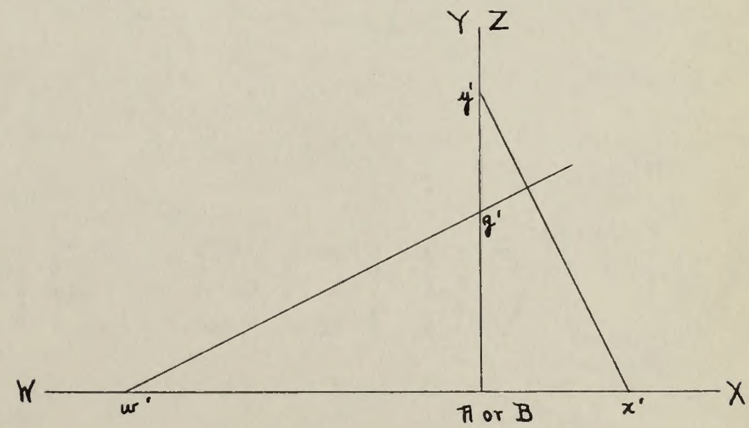


Figure XII

2. Equations of the form $P(x)-Q(y)=R(z)-S(w)$

(a) Parallel index lines.

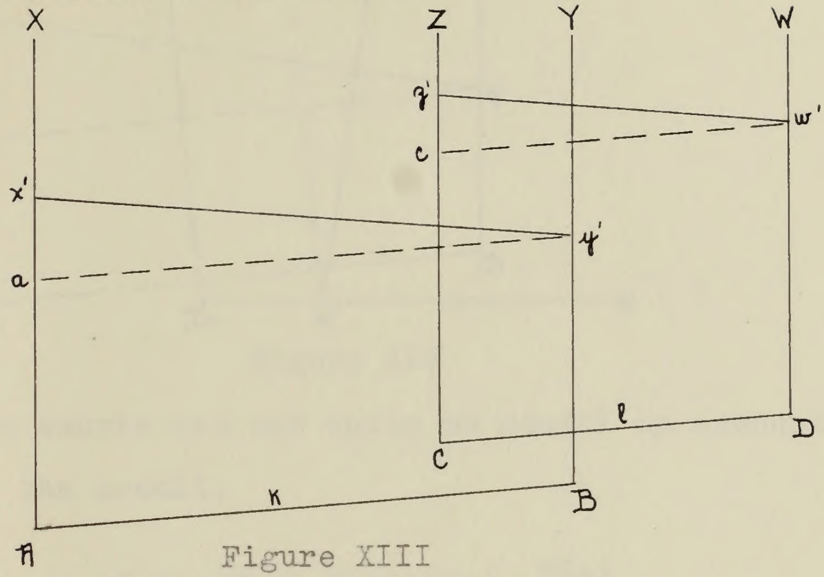


Figure XIII

Draw BY, CZ, and DW parallel to AX also making CD parallel to AB. Place $m'P(x)$ on AX, $m'Q(y)$ on BY, $m''R(z)$ on CZ, and $m''S(w)$ on DW. Let $AB = k$ and $CD = l$.

If $m':k = m'':l$

$m'P(x')-m'Q(y'):k = m''R(z')-m''S(w'):l$

Draw y'a parallel to AB and w'c parallel to CD.

$Ax'-Aa:ay' = Cz'-Cc:cw'$

Therefore the triangles ax'y' and cz'w' are similar and the index lines are parallel.

(b) Perpendicular index lines.

If CZ and DW are perpendicular to AX and BY, and CD is perpendicular to AB, the triangles are similar as before and the index lines are perpendicular.

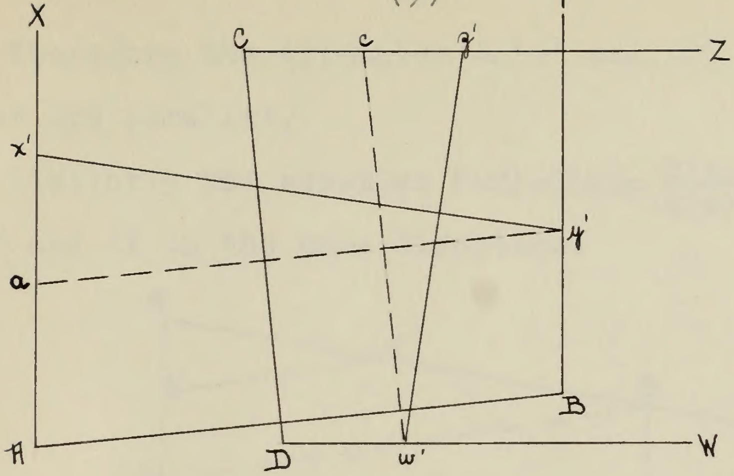


Figure XIV

These charts are not quite so useful on account of the restriction of the moduli.

3. Equations of the form $P(x)+Q(y)=\frac{R(z)}{S(w)}$

(a) Parallel index lines.

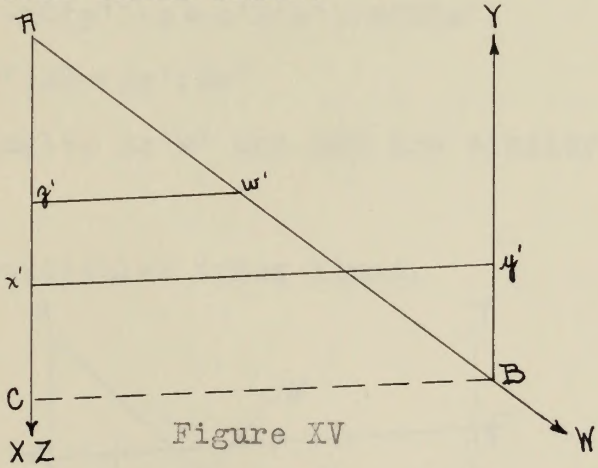


Figure XV

Draw BY parallel to AZ and in the opposite direction. Let AZ coincide with AX and AW coincide with AB. Place mP(x) on AX, mQ(y) on BY, m'R(z) on AZ, and m"S(w) on AW. Let AB = k. Draw BC parallel to x'y'.

If $m:k = m':m''$

$mP(x')+mQ(y'):k = m'R(z'):m''S(w')$

$Ax'+By':AB = Az':Aw'$

Therefore the triangles Az'w' and ACB are similar and the index lines are parallel.

Similarly the equation $P(x)-Q(y)=\frac{R(z)}{S(w)}$ can be charted by drawing BY and AX in the same direction.

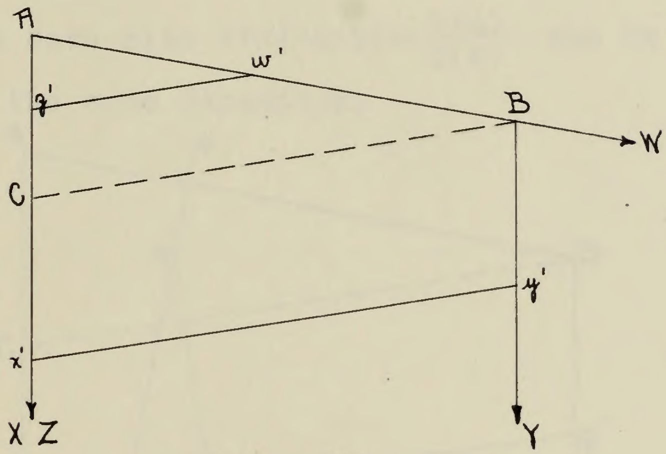


Figure XVI

$mP(x')-mQ(y'):k = m'R(z'):m''S(w')$

$Ax'-By':AB = Az':Aw'$

The triangles Az'w' and ACB are similar and the index lines are parallel.

(b) Perpendicular index lines.

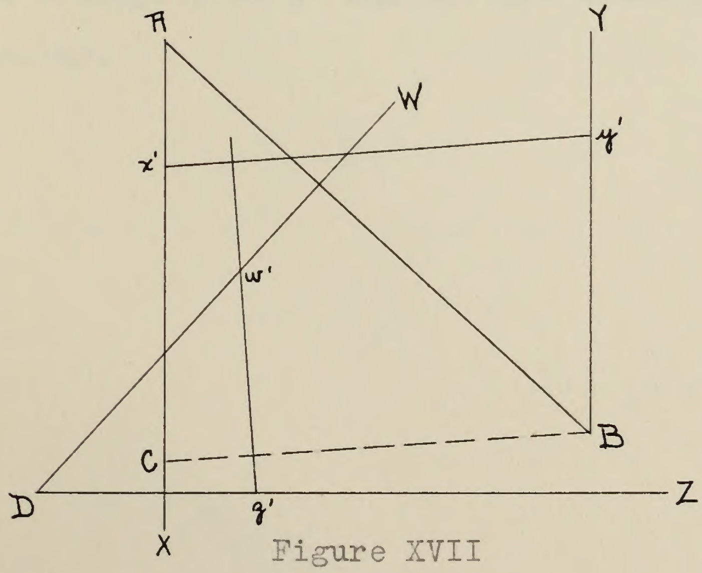


Figure XVII

Draw DZ perpendicular to AX and DW perpendicular to AB.

$Ax' + By' : AB = Dz' : Dw'$

Therefore the triangles Dz'w' and ACB are similar and the index lines are perpendicular.

In this case also $P(x) - Q(y) = \frac{R(z)}{S(w)}$ can be charted by drawing BY and AX in the same direction.

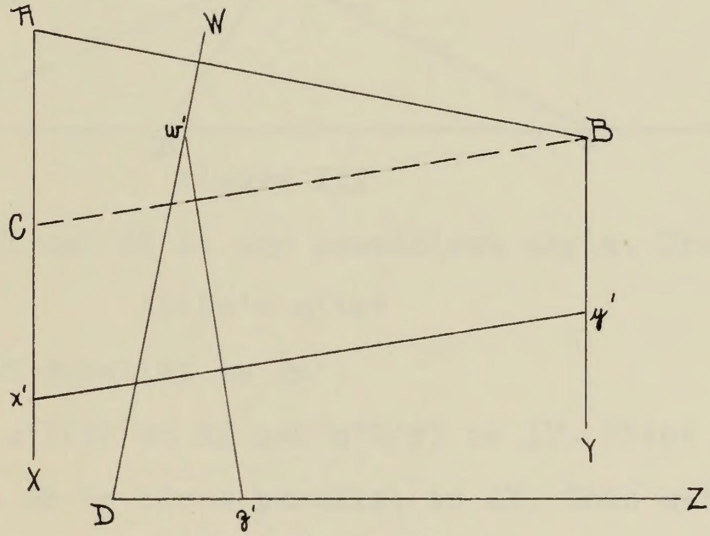


Figure XVIII

$Ax' - By' : AB = Dz' : Dw'$

The triangles Dz'w' and ACB are similar and the index lines are perpendicular.

IV. Concurrent scales.

Equations of the form $\frac{1}{P(x)} + \frac{1}{Q(y)} = \frac{1}{R(z)}$

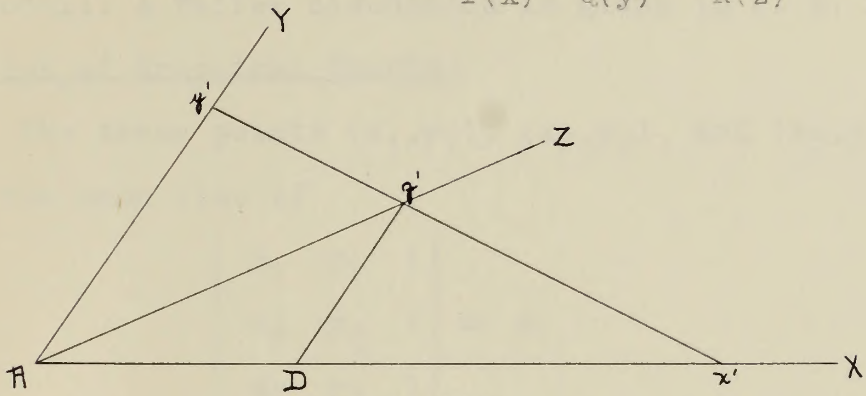


Figure XIX

Draw AX and AZ at any convenient angle. Draw Dz' such that

$$AD:Dz' = m':m''$$

Draw AY parallel to Dz'.

Place $m'P(x)$ on AX and $m''Q(y)$ on AY. Place $m'R(z)$ on AX and project onto AZ by lines parallel to AY. This makes $AD = m'R(z')$.

$$Dz':Ay' = Ax'-AD:Ax'$$

$$\frac{m''}{m'} AD:Ay' = Ax'-AD:Ax'$$

$$\frac{m''AD}{m'Ay'} = 1 - \frac{AD}{Ax'} \qquad \frac{m''AD}{m'Ay'} + \frac{AD}{Ax'} = 1$$

$$\frac{m''}{Ay'} + \frac{m'}{Ax'} = \frac{m'}{AD}$$

Since $Ay' = m''Q(y')$, $Ax' = m'P(x')$, and $AD = m'R(z')$

$$\frac{1}{P(x')} + \frac{1}{Q(y')} = \frac{1}{R(z')}$$

Therefore x', y', and z' are corresponding values of the variables.

This method can be extended to more variables.

V. The use of determinants.

Determinants can be used to find the positions of the axes and the moduli. A fuller discussion is given in J. B. Peddle's Construction of Graphical Charts.

The three points (x_1,y_1), (x_2,y_2), and (x_3,y_3) are known to be on the same line if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

If the equation to be charted is written in determinant form so that only one variable occurs in each column and then if a column of ones is obtained, the result can be interpreted in Cartesian coordinates.

The general method is illustrated by the following case:

$$LH^2 = S$$

$$\log L + 2\log H = \log S$$

$$\text{Let } \log L = x \qquad x - \log L = 0$$

$$\log H = y \qquad y - \log H = 0$$

$$x + 2y - \log S = 0$$

$$\begin{vmatrix} 1 & 0 & -\log L \\ 0 & 1 & -\log H \\ 1 & 2 & -\log S \end{vmatrix} = 0$$

Adding the second column to the first and multiplying the third column by -1,

$$\begin{vmatrix} 1 & 0 & \log L \\ 1 & 1 & \log H \\ 3 & 2 & \log S \end{vmatrix} = 0$$

Dividing the third row by 3,

$$\begin{vmatrix} 1 & 0 & \log L \\ 1 & 1 & \log H \\ 1 & \frac{2}{3} & \frac{\log S}{3} \end{vmatrix} = 0$$

The last two columns can be interpreted as the x and y of Cartesian coordinates.

That is, when $x = 0,\ y = \log L$

$x = 1,\ y = \log H$

$x = \frac{2}{3},\ y = \frac{\log S}{3}$

Since the sign of y is positive in each case, the scales should all be on the same side of the base line. The 1 and $\frac{2}{3}$ tell the relative spacing of the scales.

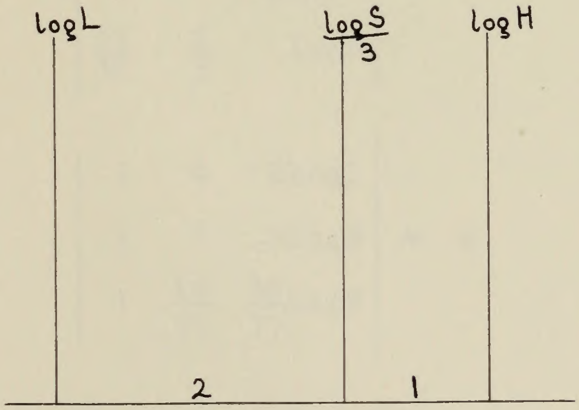


Figure XX

Suppose L is to vary from 0 to 30 and H from 0 to 100. LogL will vary from 0 to 1.477 and logH from 0 to 2. If the modulus 6 is used for logL and the modulus 5 for logH, the chart will lie within ten inches.

Going back to the original determinant, if the first column

is divided by 6, and the second column by 5, the value of the determinant is unchanged.

$$\begin{vmatrix} \frac{1}{6} & 0 & \log L \\ 0 & \frac{1}{5} & \log H \\ \frac{1}{6} & \frac{2}{5} & \log S \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & 6\log L \\ 0 & 1 & 5\log H \\ \frac{1}{6} & \frac{2}{5} & \log S \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & 6\log L \\ 1 & 1 & 5\log H \\ \frac{17}{30} & \frac{2}{5} & \log S \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 0 & 6\log L \\ 1 & 1 & 5\log H \\ 1 & \frac{12}{17} & \frac{30}{17}\log S \end{vmatrix} = 0$$

This means that when $x = 0$, $y = 6\log L$

$x = 1$, $y = 5\log H$

$x = \frac{12}{17}$, $y = \frac{30}{17}\log S$

The unit on the base line does not need to be the same as that for the scales. The ratio of the segments is all that is determined.

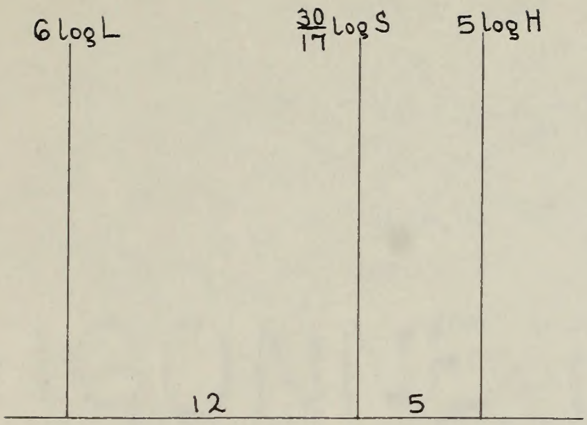


Figure XXI

This is the same result as would be obtained by letting $m' = 6$ and $m'' = \frac{5}{2}$.

$$\frac{m'm''}{m'+m''} = \frac{15\cdot 2}{17} = \frac{30}{17}$$

but the method is general and can be used for other types of equations.

B. Applications

I. Gases.

a. Boyle's law. (Chart I, page 19)

The pressure of a gas varies inversely as the volume at constant temperature.

$$P = \frac{k}{V} \qquad P' = \frac{k}{V'}$$

$$P : P' = V' : V$$

$$m'P : m''P' = n'V' : n''V \quad \text{where } m' : m'' = n' : n''$$

Let $m' = m''$ and $n' = n''$ in order to have one scale for V and V' and one for P and P'.

If the chart includes values of P from 0 to 1,000 mm. and values of V from 0 to 100 liters, with 1 mm. as unit and $m' = m'' = .2$ and $n' = n'' = 2$, the scales will be 20 cm. long.

<u>To read the chart</u> connect P with V' or P' with V to determine the intersection of the index lines with AB. The value of the unknown will be determined by the other index line.

Either scale can be interpreted in any units that are desirable. Therefore in dealing with small quantities of gas, if the scale is interpreted in cubic centimeters, the scale becomes more spread out and the result can be obtained more accurately.

The graph of this function would be a hyperbola for any given temperature for a certain gas. This chart can be used for any value of k. It can also be used equally well for any problem in which one factor is inversely proportional to another. It is known that "the depth of solution in a colorimeter is inversely proportional to the concentration of the colored material. The conductance of

a solution is equal to the reciprocal of its resistance. The product of the concentration of the hydrogen ions and the hydroxyl ions in an aqueous solution is a constant." [1]

b. Charles' law. (Chart II, page 20)

At constant pressure the volume of a gas varies directly with the absolute temperature.

$$V = kT \qquad V' = kT'$$

$$V : V' = T : T'$$

This chart is very similar to the preceding one. In this case corresponding values of the variables are connected to find the intersection of the index lines with AB.

If C is placed at 273, then the scale can be marked with the centigrade temperatures.

These charts can be read by placing a sheet of transparent paper with a line drawn on it so that the line serves as the first index line. The unknown value can be found by placing a straightedge (or the edge of a triangle) through the other given value and the intersection on AB.

1. F. Daniels, Mathematical preparation for physical chemistry, page 54.

Chart I

Boyle's law

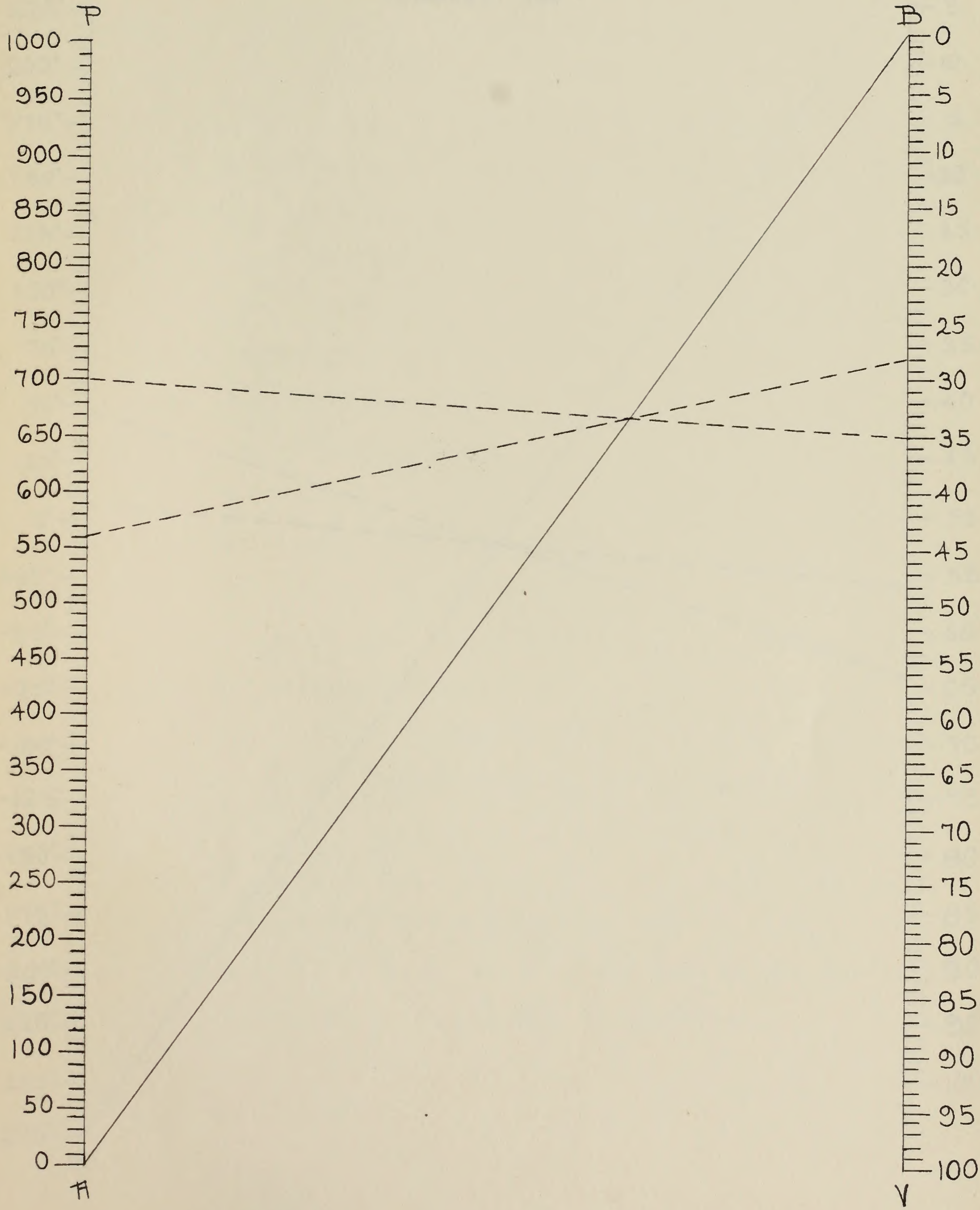
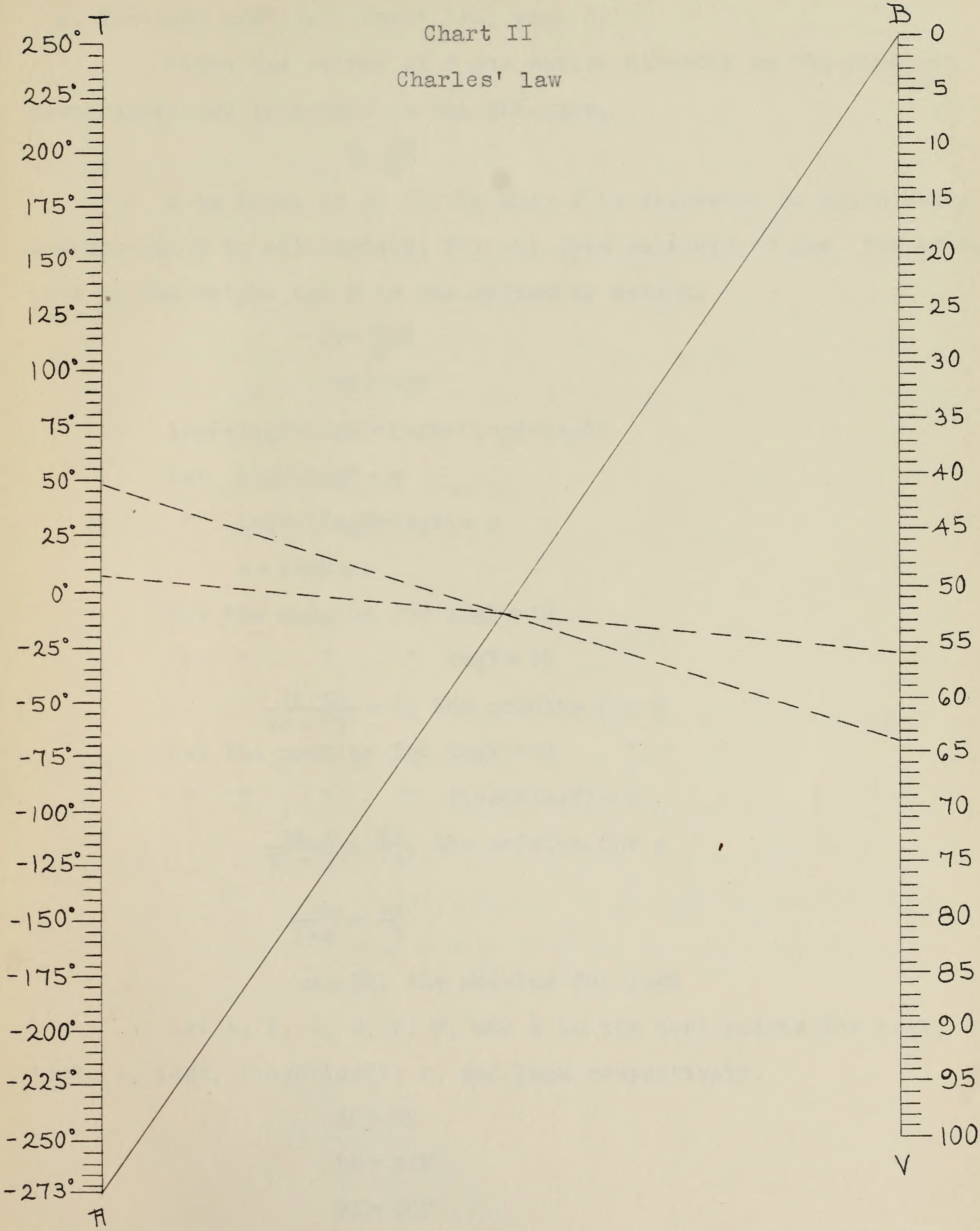


Chart II

Charles' law



c. Combined gas law. (Chart III, page 23)

Since the volume of a gas varies directly as the absolute temperature and inversely as the pressure,

$$P = \frac{RT}{V}$$

R is found to be 62,380 when V is expressed in cubic centimeters and P in millimeters, for one gram molecule of gas. Therefore if W is the weight and M is the molecular weight,

$$PV = \frac{W}{M}RT$$

$$PVM = WRT$$

$$\log P + \log V + \log M = \log W + (\log R + \log T)$$

Let $\log P + \log V = q$

" $\log W + (\log R + \log T) = r$

$q + \log M = r$

Let the modulus for $\log P = 10$

" " " " $\log V = 10$

$\frac{10 \cdot 10}{10 + 10} = 5$, the modulus for q

Let the modulus for $\log W = 10$

" " " " $(\log R + \log T) = 5$

$\frac{10 \cdot 5}{10 + 5} = \frac{10}{3}$, the modulus for r

$$\frac{5m}{5+m} = \frac{10}{3}$$

$m = 10$, the modulus for $\log M$

Let A, B, C, D, E, F, and G be the zero points for logP, logV, q, logW, (logR+logT), r, and logM respectively.

$$AC = BC$$

$$DF = 2EF$$

$$FG = 2CF$$

The characteristics can be disregarded in making the scales if the position of the decimal point is determined by some other means. In general its position is evident from the nature of the problem. The P and W scales can be made to coincide. This and the V and M scales are easily constructed, being logarithmic scales with the modulus 10. With any logarithmic scale, the scale with any modulus can be obtained by projection. The points on the T scale can be found by projecting from G the points on the V scale and then adding 5logR. This scale should be repeated so that the entire line will be marked.

<u>To read the chart</u> draw any two of the three following index lines which are determined by the given values.

PVq'

WTr'

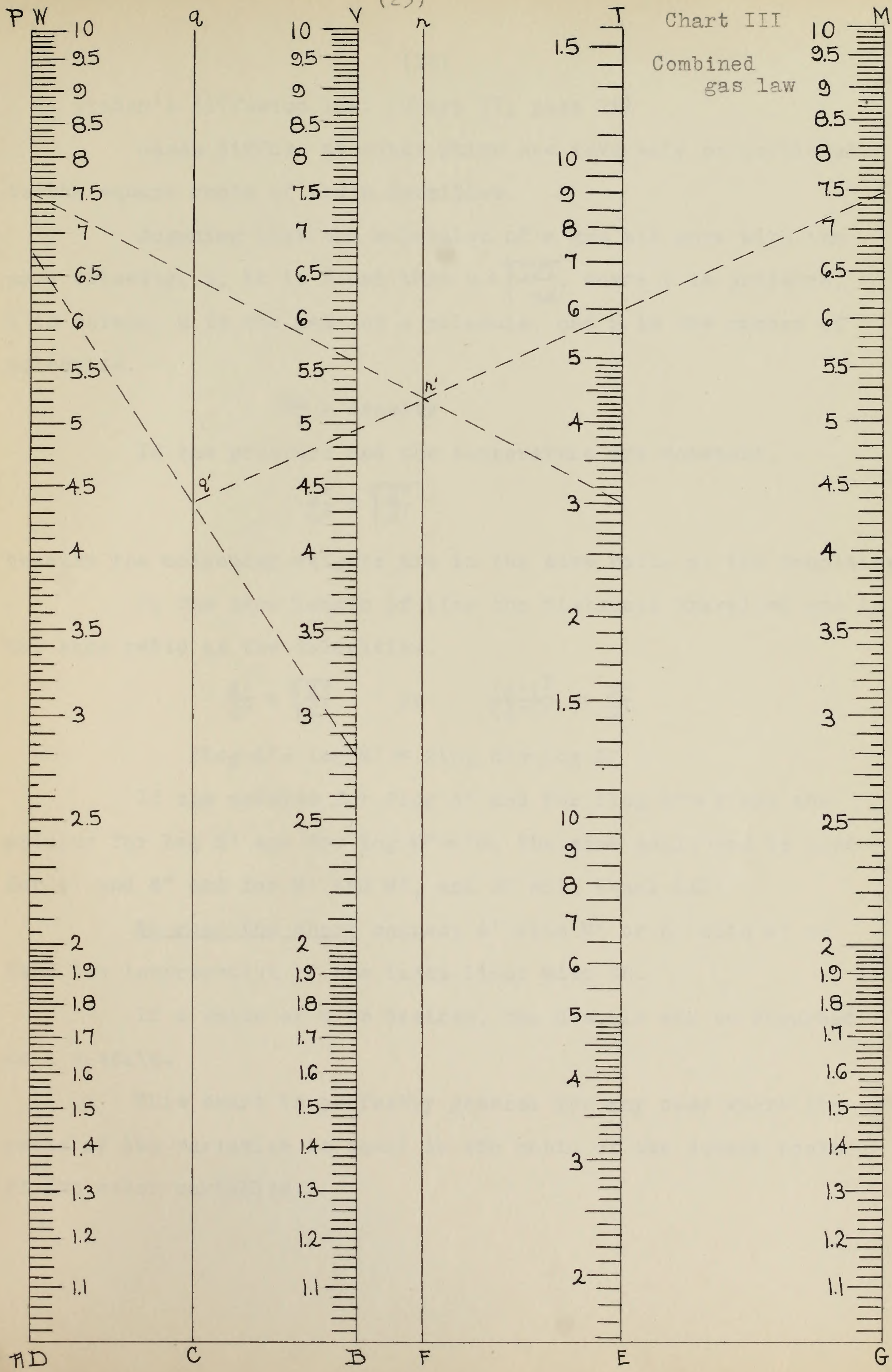
q'r'M

The unknown will be determined by the third index line. The lines should be drawn on transparent paper.

The position of the decimal point should be disregarded in reading the logarithmic scales. They are read as logarithm tables or a slide rule. To avoid determining the characteristic, the logarithmic scales can be marked with definite values, thereby limiting the range of the chart.

Chart III

Combined gas law



d. Graham's diffusion law. (Chart IV, page 25)

Gases diffuse at rates which are inversely proportional to the square roots of their densities.

Assuming that the molecules of a gas all move with the same velocity, u, it is found that $u = \sqrt{\frac{3PV}{mn}}$, where P is pressure, V is volume, m is the mass of a molecule, and n is the number of molecules.

$$\frac{mn}{V} = \text{density}$$

If the pressure and the temperature are constant,

$$\frac{u'}{u''} = \sqrt{\frac{M''}{M'}}$$

because the molecular weights are in the same ratio as the densities.

In the same length of time the distances travelled are in the same ratio as the velocities.

$$\frac{d'}{d''} = \sqrt{\frac{M''}{M'}} \qquad \text{or} \qquad \frac{(d')^2}{(d'')^2} = \frac{M''}{M'}$$

$$2\log d' + \log M' = 2\log d'' + \log M''$$

If the modulus for 2log d' and for 2log d" = 5 and the modulus for log M' and for log M" = 10, the same scale can be used for d' and d" and for M' and M", and BC will equal 2AC.

<u>To read the chart</u> connect d' with M' or d" with M" to find the intersection of the index lines with CD.

If a value of u is desired, the d-scale can be regarded as a u-scale.

This chart is perfectly general for any case where the ratio of two variables is equal to the ratio of the square roots of two other variables.

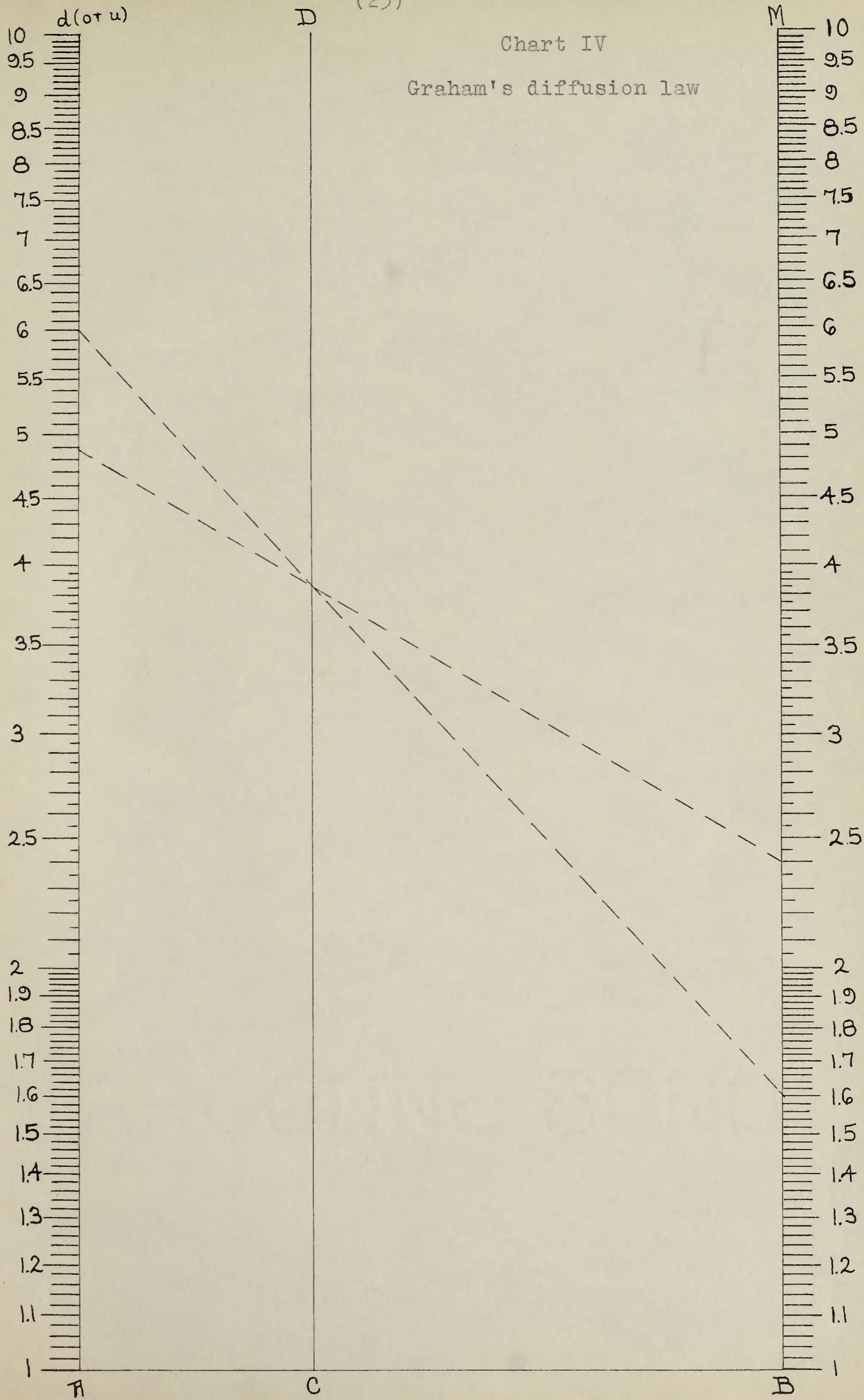


Chart IV

Graham's diffusion law

e. Isothermal expansion. (Chart V, page 28)

If a gas expands isothermally, there is no change in its internal energy.

$$\Delta U = 0$$

Since $\Delta U = q - w$, where q is the heat absorbed and w is the work done, it follows that $q = w$.

If V' is the initial volume and V" is the final volume,

$$w = \int_{V'}^{V''} P dV$$

Assuming that $PV = RT$,

$$w = \int_{V'}^{V''} RT\frac{dV}{V} = RT \ln \frac{V''}{V'} = RT \ln \frac{P'}{P''}$$

If V is expressed in liters and P is expressed in atmospheres, $R = .082$.

$$\frac{w}{T} = \frac{\log V'' - \log V'}{\frac{.4343}{.082}}$$

Let $\log V'' - \log V' = s$

$$\log V'' = s + \log V'$$

$$\frac{w}{T} = \frac{s}{5.3}$$

$$\frac{m'w}{m''T} = \frac{n's}{n''5.3}$$

Let $m' = .6$, $m'' = .04$, and $n'' = \frac{1}{3}$

$$n' = \frac{.6}{.04} \cdot \frac{1}{3} = \frac{.6}{.12} = 5$$

Let the modulus for $\log V' = 10$

$$\frac{5 \cdot 10}{5 + 10} = \frac{10}{3},$$ the modulus for $\log V''$

$$\frac{.6w}{.04T} = \frac{5s}{1.8}$$

Make AC = 2BC and project the points from the V' scale from B as vertex on the V" scale. The w and T scales can be on the same line if 1.8" is marked off on the s-scale. A centimeter rule can be used for the w and T scales to increase the range, since it is only necessary to use the same unit for these two.

To read the chart connect 1.8 with T and draw either of the two index lines determined by the given values,

V'V"s'

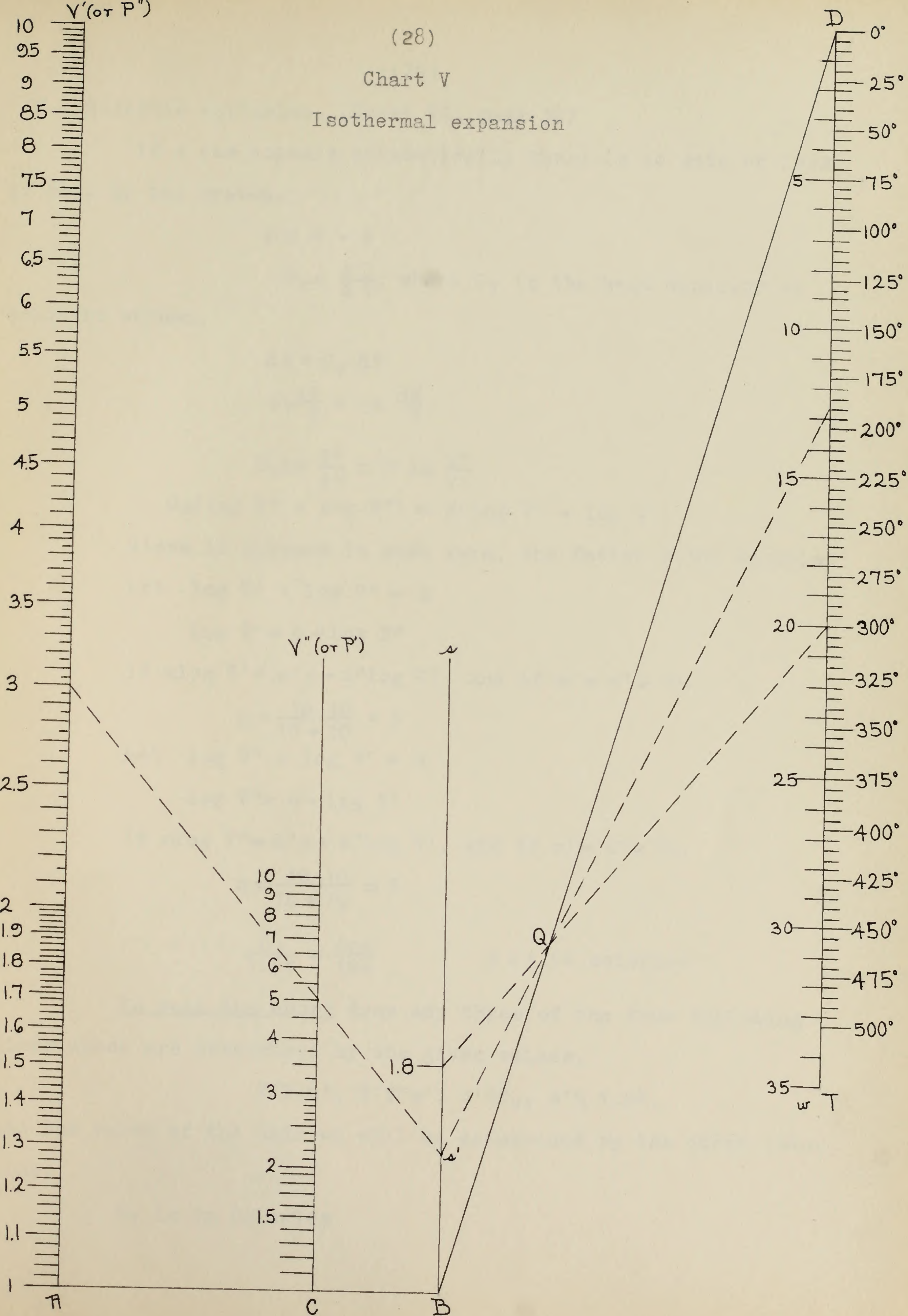
wQs'

and the value of the unknown will be determined by the other index line.

w is in atmosphere-liters

Chart V

Isothermal expansion



f. Adiabatic expansion. (Chart VI, page 30)

If a gas expands adiabatically there is no gain or loss of heat by the system.

$$\Delta U = - w$$

$C_V = \frac{\Delta U}{\Delta T}$, where C_V is the heat capacity at constant volume.

$$\Delta U = C_V \cdot \Delta T$$

$$C_V \frac{dT}{T} = -R \frac{dV}{V}$$

$$C_V \ln \frac{T'}{T''} = R \ln \frac{V''}{V'}$$

$$C_V(\log T' - \log T'') = R(\log V'' - \log V')$$

Since ln appears in each term, the factor 2.303 cancels.

Let $\log T' - \log T'' = s$

$\log T' = s + \log T''$

If $m \log T' = m's + m'' \log T''$, and if $m' = m'' = 10$,

$$m = \frac{10 \cdot 10}{10 + 10} = 5$$

Let $\log V'' - \log V' = u$

$\log V'' = u + \log V'$

If $n \log V'' = n'u + n'' \log V'$, and if $n' = n'' = 10$,

$$n = \frac{10 \cdot 10}{10 + 10} = 5$$

$$\frac{C_V}{1.98} = \frac{10u}{10s} \qquad R = 1.98 \text{ calories}$$

<u>To read the chart</u> draw any three of the four following lines which are determined by the given values,

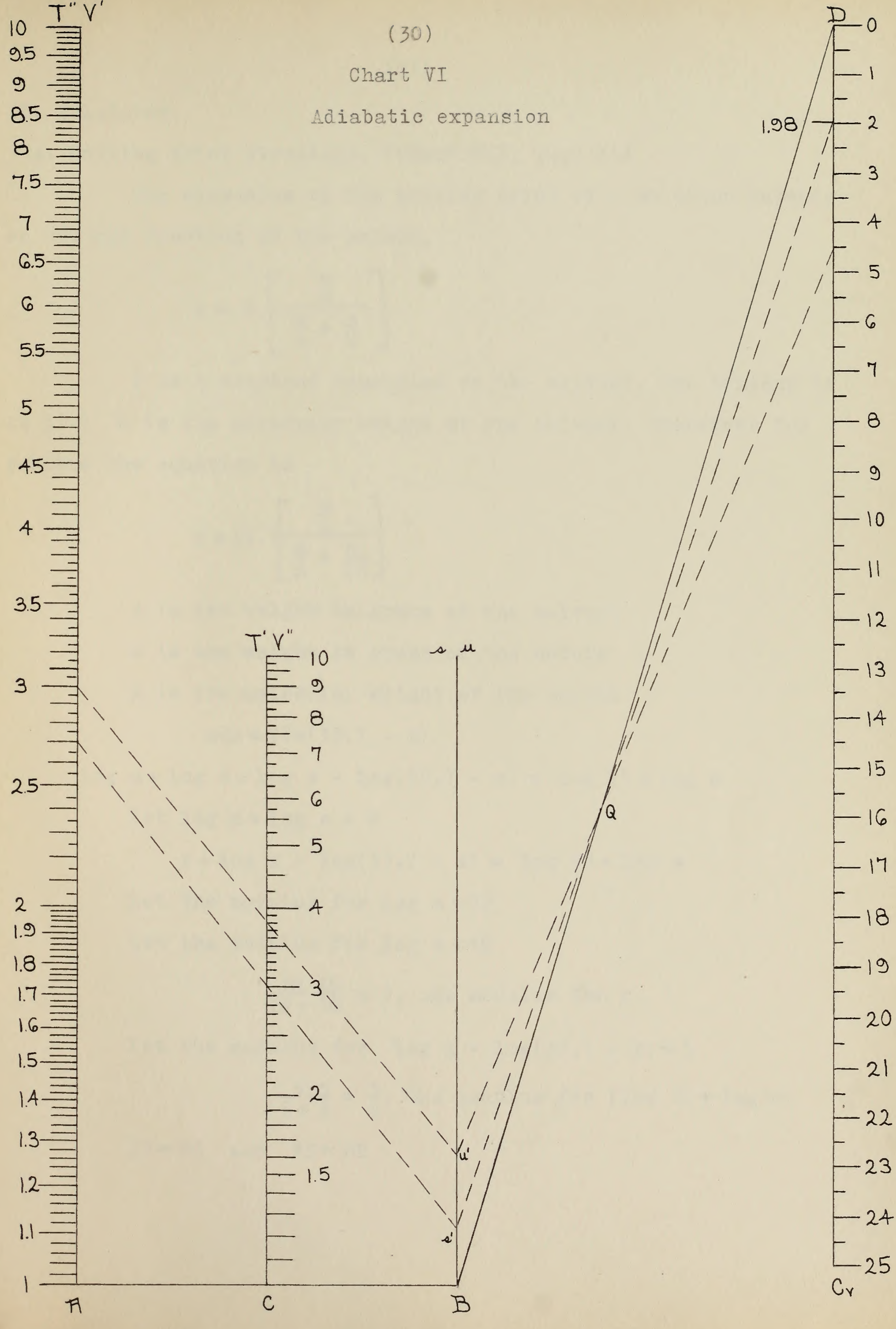
$T''T'u'$, $V'V''s'$, $u'QC_V$, $s'Q\ 1.98$,

and the value of the unknown will be determined by the other index line.

C_V is in calories

Chart VI

Adiabatic expansion



II. Solutions.

a. Boiling point elevation. (Chart VII, page 33)

The elevation of the boiling point of a solution depends on the mol fraction of the solute.

$$x = B\left[\frac{\frac{w}{m}}{\frac{w}{m}+\frac{s}{M}}\right]$$

B is a constant depending on the solvent. For benzene it is 35.7 M is the molecular weight of the solvent. Therefore for benzene the equation is

$$x = 35.7\left[\frac{\frac{w}{m}}{\frac{w}{m}+\frac{s}{78}}\right]$$

s is the weight in grams of the solvent

w is the weight in grams of the solute

m is the molecular weight of the solute

$$msx = 78w(35.7 - x)$$

$$\log m + \log s + \log x - \log(35.7 - x) = \log 78 + \log w$$

Let $\log m + \log s = r$

$$r + \log x - \log(35.7 - x) = \log 78 + \log w$$

Let the modulus for $\log m = 10$

Let the modulus for $\log s = 10$

$$\frac{10 \cdot 10}{10 + 10} = 5\text{, the modulus for } r$$

Let the modulus for $\log x - \log(35.7 - x) = 5$

$$\frac{5 \cdot 5}{5 + 5} = \frac{5}{2}\text{, the modulus for } (\log 78 + \log w)$$

$AB = BC$ and $BD = DE$

The characteristics were disregarded and the w-scale repeated to cover the entire line. The position of w to be used in reading the chart will be evident from the nature of the problem and a comparison with the normal boiling point elevation. The values on the m, s, and w scales can be marked so as to take account of the characteristics, but this necessarily narrows the range of values for which the chart can be used.

To read the chart connect m and s to find r'. The value of x will be determined by the other index line r'w.

A similar chart can be made for any other solvent by substituting the proper constants for M and B.

b. Freezing point lowering.

Charts for the freezing point lowering of solutions would be made exactly like the preceding with the freezing point constant, F, in place of B.

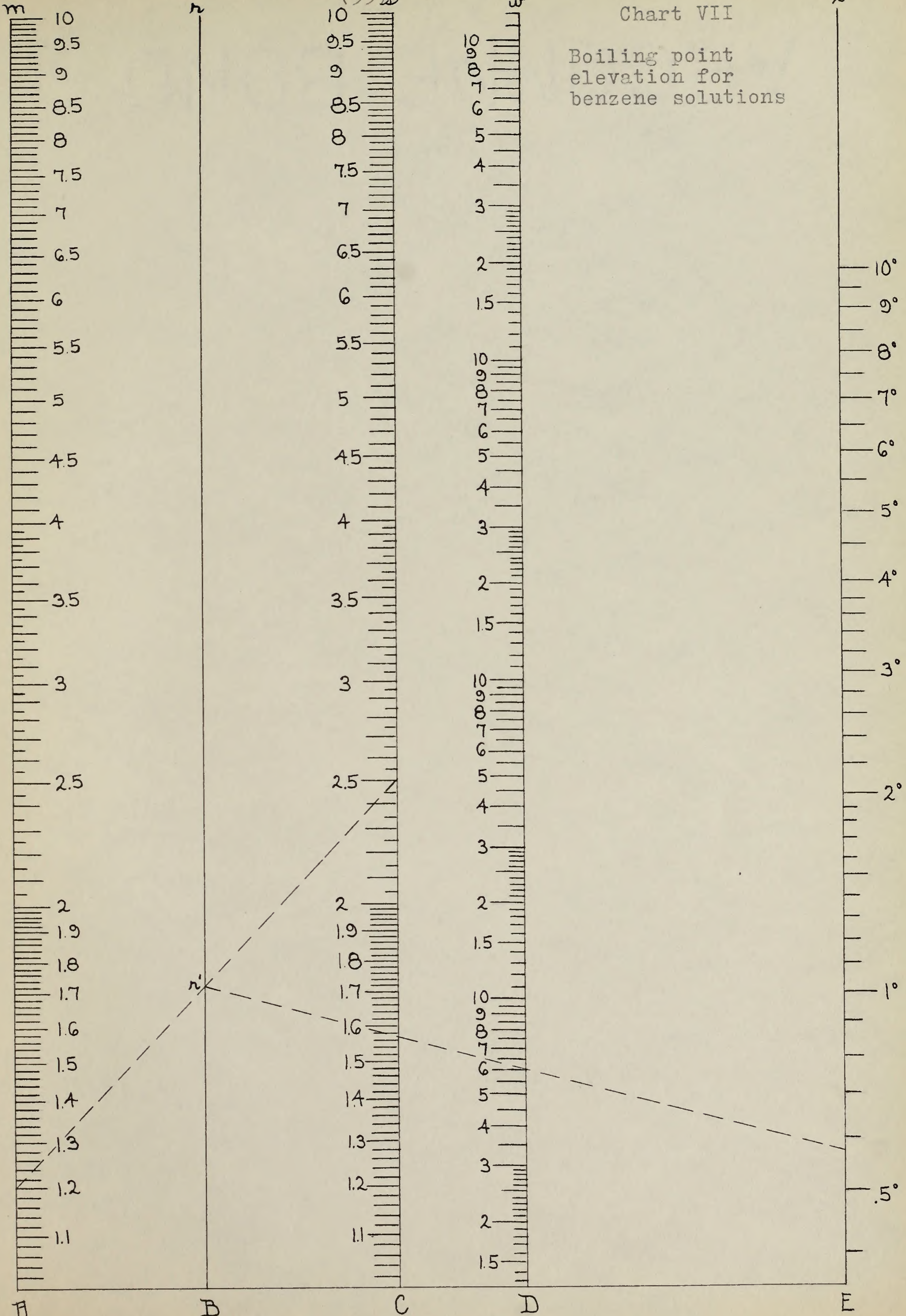


Chart VII

Boiling point elevation for benzene solutions

III. First order reaction. (Chart VIII, page 35)

In a first order or monomolecular reaction the reaction velocity is proportional to the concentration of the reacting substance. If A is the original concentration and x is the amount transformed at time t,

$$\frac{dx}{dt} = k(A-x)$$

$$\frac{dx}{A-x} = k\,dt$$

$$-\ln(A-x) = kt + c$$

When $t = 0$, $x = 0$, therefore $c = -\ln A$

$$kt = \ln\frac{A}{A-x} \qquad \text{or} \qquad kt = 2.303 \log\frac{A}{A-x}$$

If $f = \frac{x}{A}$, the fraction transformed,

$$k = \frac{2.303}{t}\log\frac{1}{1-f}$$

$$\frac{k}{2.303} = \frac{\text{colog}(1-f)}{t}$$

$$\frac{10{,}000k}{4\cdot 2.303} = \frac{5\ \text{colog}(1-f)}{.002t}$$

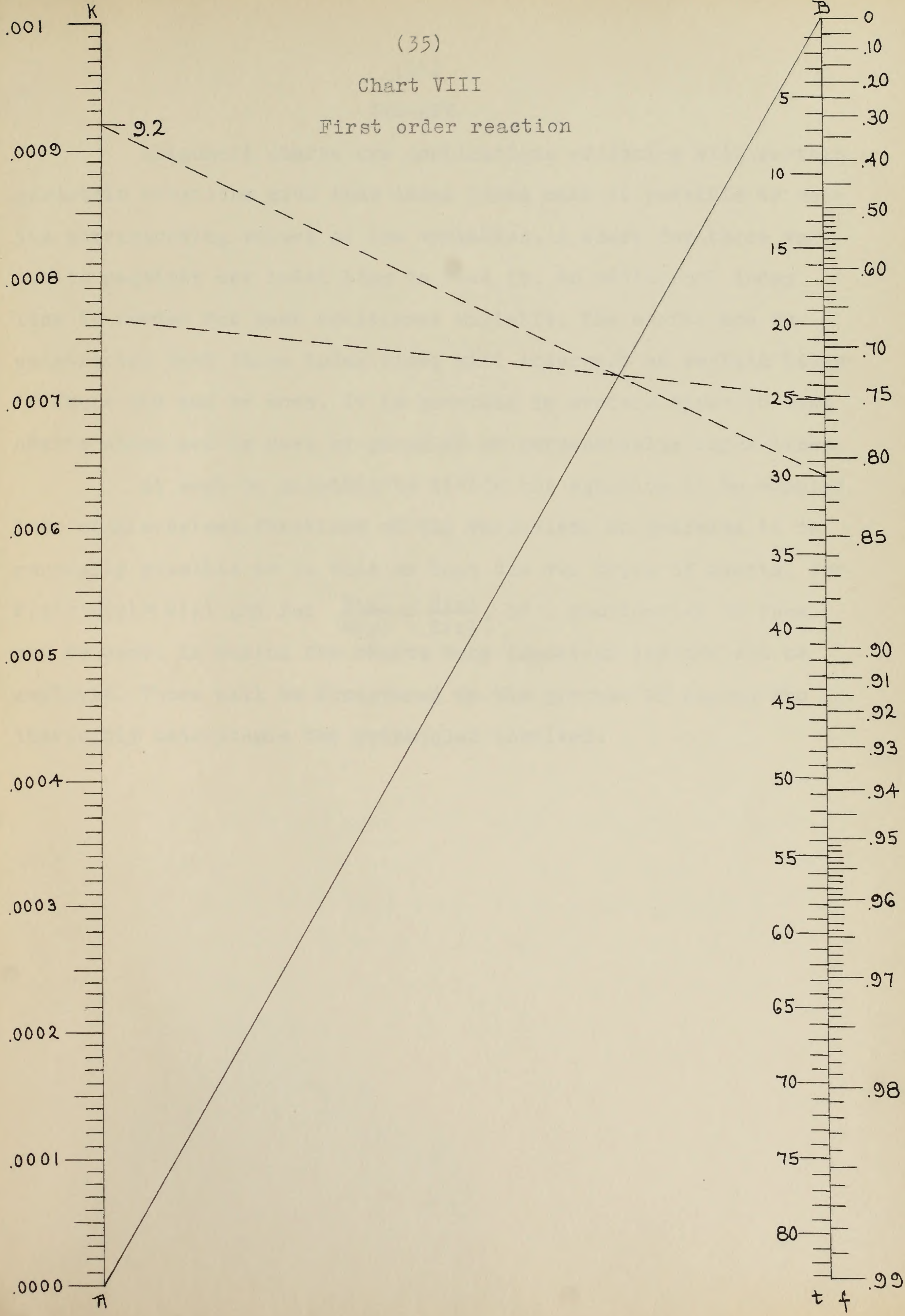
The modulus 5 was used for colog(1-f) in order to carry f to 99%. If k is expressed in moles per liter per second, one second is .002" or one minute is .12".

To read the chart connect k (in moles per liter per second) with f, or connect 9.2 with t (in minutes), and the other index line will determine the value of the unknown.

In a bimolecular reaction if the concentration of one substance is practically constant, as the concentration of water in the hydrolysis of sugar, the reaction is treated as of the first order. The range of the chart is for this type of reaction. Charts for real first order reactions can be made by using different moduli.

Chart VIII

First order reaction



SUMMARY

Alignment charts are combinations of scales with certain geometric relations such that index lines make it possible to read the corresponding values of the variables. A chart for three variables requires one index line to read it. An additional index line is needed for each additional variable. The charts are so constructed that these index lines will intersect on certain lines if there are two or more. It is possible in certain cases to make charts which can be read by parallel or perpendicular index lines.

It must be possible to divide the equation to be charted into single-valued functions of the variables. In practice it is generally possible to do this so that the two types of charts, for $P(x) + Q(y) = R(z)$ and for $\frac{P(x)}{Q(y)} = \frac{R(z)}{S(w)}$, or a combination of them can be used. In making the charts many ingenious devices can be employed. These will be discovered in the process by anyone who thoroughly understands the principles involved.

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Notes were also used from the following two courses at Boston University.

Notes on Alignment Charts from the course in Curve Tracing given by Professor R. E. Bruce 1927-28.

Notes from the lecture course in Physical Chemistry given by Professor E. O. Holmes 1928-29.

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